Hardy's Nonlocality Argument as a Witness for Post-Quantum Correlations

(DIQIP/QALGO meeting)

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- Motivation is to search for Physical Principle(s) that can separate quantum from post-quantum correlations; ideally reproduce the exact set of quantum correlations.
- Proposed principles like, Non-trivial Communication Complexity, Macroscopic Locality, Information Causality, Local-Orthogonality, has produced some promising results.

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- Recently, it has been shown that information principles which are applicable in a non-trivial way only to bi-partite scenarios, like communication complexity and information causality, are insufficient to distinguish certain correlations when three or more parties are involved (Gallego et al., 2011; Das et al. 2013).

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- This in turn has lead to search for generalized multiparty information principles for characterization of quantum correlations, formulation of Local-Orthogonality principle is a recent new proposal in this direction (Fritz et al., 2013).
- Also, study of multiparty nonlocal correlations has lead to its more precise characterization (Gallego et al., 2012; Bancal et al., 2013).

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Quantum Correlations Require Multipartite Information Principles

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Identifying which correlations among distant observers are possible within our current description of nature, based on quantum mechanics, is a fundamental problem in physics. Recently, information concepts have been proposed as the key ingredient to characterize the set of quantum correlations. Novel information principles, such as information causality or nontrivial communication complexity, have been introduced in this context and successfully applied to some concrete scenarios. We show in this work a fundamental limitation of this approach: no principle based on bipartite information concepts is able to singleout the set of quantum correlations for an arbitrary number of parties. Our results reflect the intricate structure of quantum correlations and imply that new and intrinsically multipartite information concepts are needed for their full understanding.

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Hardy's nonlocality argument as a witness for postquantum correlations

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Recently, Gallego *et al.* [Phys. Rev. Lett. **107**, 210403 (2011)] proved that any future information principle aiming at distinguishing between quantum and postquantum correlation must be intrinsically multipartite in nature. We establish similar results by using the device-independent success probability of Hardy's nonlocality argument for tripartite quantum systems. We construct an example of a tripartite Hardy correlation which is postquantum but satisfies not only the all-bipartite information principle but also the guess-your-neighbor's-input (GYNI) inequality.

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- First we proved a device independent bound for success probability of a tri-partite Hardy's nonlocality argument; violation of such a bound demonstrate correlations to be post-quantum.
- Then, we constructed an example of Time-Ordered-Bi-Local (TOBL) correlations which shows Hardy's nonlocality, but, with a success probability exceeding the derived device independent bound.

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- Then, we constructed an example of Time-Ordered-Bi-Local (TOBL) correlations which shows Hardy's nonlocality, but, with a success probability exceeding the derived device independent bound.
- Since TOBL correlations satisfy any bi-partite information principle, hence we show insufficiency of such principles in distinguishing all post-quantum correlations.

Hardy-Nonlocality argument in tripartite scenario...

Consider three spatially separated parties, say, Alice, Bob and Charlie. They respectively choose to input (measure) X_i, Y_j, Z_k where $i, j, k \in \{0, 1\}$. Any input can have binary outcome ± 1. Suppose 5 out 64 joint probabilities $P(a, b, c|X_i, Y_j, Z_k)$ satisfy the following restrictions:

$$P(+++|X_0\ Y_0\ Z_0) > 0 \quad (1)$$
$$P(+++|X_1\ Y_0\ Z_0) = 0 \quad (2)$$
$$P(+++|X_0\ Y_1\ Z_0) = 0 \quad (3)$$
$$P(+++|X_0\ Y_0\ Z_1) = 0 \quad (4)$$
$$P(---|X_1\ Y_1\ Z_1) = 0 \quad (5)$$

Then, there is no local-realistic description for such events.

Results for success probability of Hardy-nonlocality...

Denote, $p_H > 0$ as the success probability of the Hardy's argument.

- In quantum mechanics, for three qubit system subject to local projective measurements, $(p_H)_{max} = \frac{1}{8}$ (Chaudhary et al. 2010).
- We showed that $(p_H)_{max} = \frac{1}{8}$ is in fact a device independent number, i.e., independent of the local Hilbert space dimension and type of measurements performed.

Derivation of device independent bound...

- At Alice's end, X_0, X_1 being hermitian operators with eigen values ± 1, $X_0 = \prod_{+|X_0} - \prod_{-|X_0}$; $X_1 = \prod_{+|X_1} - \prod_{-|X_1}$.
- A Lemma proved by Masanes PRL(2006), the Hibert space H can be decomposed as a direct sum of subspace H^i of dimensions atmost two such that $X_0 = \oplus_i X_0^i$ and $X_1 = \oplus_i X_1^i$, where X_0^i and X_1^i acts within H^i.
- Same is true at Bob's and Charlie's ends, therefore,

$$\begin{aligned} P(abc|x,y,z) &= \mathrm{Tr}(\rho \prod_{a|x} \otimes \prod_{b|y} \otimes \prod_{c|z}) \\ &= \sum_{i,j,k} q_{ijk} \mathrm{Tr}(\rho_{ijk} \prod_{a|x}^{i} \otimes \prod_{b|y}^{j} \otimes \prod_{c|z}^{k}) \\ &= \sum_{i,j,k} q_{ijk} P_{ijk}(abc|xyz) \end{aligned}$$

where $q_{ijk} = \mathrm{Tr}(\rho \prod^i \otimes \prod^j \otimes \prod^k)$ and $\rho_{ijk} = \frac{1}{q_{ijk}}[\prod^i \otimes \prod^j \otimes \prod^k \rho \prod^i \otimes \prod^j \otimes \prod^k]$ is, at most, a three qubit state; $q_{ijk} \geq 0$ and $\sum_{ijk} q_{ijk} = 1$.

Derivation...

- The Hardy's condition are satisfied for P iff they are satisfied for each of P_{ijk}, thus,

$$P(+++|X_0 Y_0 Z_0) = \sum_{ijk} q_{ijk} P_{ijk}(+++|X_0 Y_0 Z_0)$$

  is a convex sum of Hardy's probabilities in each three-qubit space.

- Being a convex sum, the success probability P_H is thus less then or equal to $\frac{1}{8}$.

- $(P_H)_{max} = \frac{1}{8}$.

Time-Ordered-Bi-Local (TOBL) correlations... (Pironio et al. 2011)

A tripartite distribution $P(abc|xyz)$ admits a TOBL model if it can be decomposed for bipartisan $A|BC$ as:

$$\begin{aligned} P(abc|xyz) &= \sum_\lambda p_\lambda P(a|x,\lambda) P_{B\to C}(bc|yz,\lambda) \\ &= \sum_\lambda p_\lambda P(a|x,\lambda) P_{B\leftarrow C}(bc|yz,\lambda) \end{aligned}$$

and similarly for other bipartisan $B|AC$ and $C|AB$. Additionally, following conditions must hold:

$$P_{B\to C}(b|y,\lambda) = \sum_c P_{B\to C}(bc|yz,\lambda)$$
$$P_{B\leftarrow C}(c|z,\lambda) = \sum_b P_{B\leftarrow C}(bc|yz,\lambda)$$

Distributions $P_{B\to C}$ and $P_{B\leftarrow C}$ can be signaling in at most one direction.

A Hardy-correlation in TOBL set...

TABLE I. Tripartite no-signaling probability distribution $P(abc|xyz)$ with Hardy's success 1/5.

	abc							
xyz	000	001	010	011	100	101	110	111
000	$\frac{1}{5}$	0	0	$\frac{1}{5}$	0	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
001	0	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{2}{5}$	0
010	0	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{2}{5}$	$\frac{1}{10}$	0
011	0	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{10}$	$\frac{1}{10}$	0
100	0	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{10}$	0
101	0	$\frac{1}{10}$	$\frac{2}{5}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{10}$	0
110	0	$\frac{2}{5}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{5}$	0
111	$\frac{2}{5}$	0	0	$\frac{1}{5}$	0	$\frac{1}{5}$	$\frac{1}{5}$	0

- $P(000|000) = 1/5$; $P(000|100) = 0$; $P(000|010) = 0$; $P(000|001) = 0$; $P(111|111) = 0$.
- Symmetric under permutation of parties.

TOBL model for the given example

TABLE II. TOBL decomposition for the case $A|B \to C$.

λ	p_λ	a_0	a_1	b_0	b_1	c_{00}	c_{01}	c_{10}	c_{11}
1	$\frac{1}{10}$	0	0	1	1	1	0	1	1
2	$\frac{1}{10}$	0	0	1	1	1	1	0	1
3	$\frac{1}{10}$	1	0	0	0	1	1	1	0
4	$\frac{1}{10}$	1	0	1	0	0	0	1	0
5	$\frac{1}{5}$	1	0	1	0	1	0	1	0
6	$\frac{1}{10}$	0	1	0	0	0	1	1	1
7	$\frac{1}{10}$	0	1	0	1	0	1	0	0
8	$\frac{1}{10}$	1	1	1	0	0	0	0	1
9	$\frac{1}{10}$	1	1	0	1	1	0	0	0

TABLE III. TOBL decomposition for the case $A|B \leftarrow C$.

λ	p_λ	a_0	a_1	b_{00}	b_{01}	b_{10}	b_{11}	c_0	c_1
1	$\frac{1}{10}$	0	0	1	0	1	1	1	1
2	$\frac{1}{10}$	0	0	1	1	0	1	1	1
3	$\frac{1}{10}$	1	0	1	1	1	0	0	0
4	$\frac{1}{10}$	1	0	0	1	0	0	1	0
5	$\frac{1}{5}$	1	0	1	1	0	0	1	0
6	$\frac{1}{10}$	0	1	0	1	1	1	0	0
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Key features of the constructed example...

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- Thus, it also respects Local-Othogonality for single copy $LO^{(1)}$ which is equivalent to the three party GYNI game.
- Can $LO^{(k)}$ for some finite $k > 1$ eliminate this post-quantum correlation? This question needs further study.

THANK YOU...